A COUNTER EXAMPLE FOR THE HARMONIC "GAUGE CONDITION"

IN

GENERAL RELATIVITY

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Abstract

It is pointed out that the applicability of the harmonic gauge has never been generally established if the resulting coordinate system is expected to be physically realizable. A gravitational plane wave is given to show that the harmonic "gauge" may not be applicable and that implications of the linearized gauge on plane waves can be invalid. Concurrently, it is shown that, for weak gravitational waves, the harmonic gauge is valid only if the Einstein tensor is of second order deviation from the flat metric. Since such an order is gauge invariant, a gravitational weak wave with an Einstein tensor of first order, implies that the harmonic gauge can be a misnomer. Moreover, it is shown that the "gauge condition" may not be compatible with coordinate relativistic causality and the equivalence principle. Concurrently, it is also shown that the Ohanian and Ruffini approach in which, different from Einstein's, linear gravity is considered as derived from their gauge theory is not valid in physics.

Key Words: harmonic gauge, weak gravity, gravitational wave, Einstein's equivalence principle, principle of causality.

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1. Introduction.

In 1917, Hilbert [1,2] shows that a solution of Einstein equation is not unique, and depends on a choice of the physical coordinates. It was believed also that the harmonic "gauge" be unconditionally applicable. In this paper, a counter example is provided to call the attention to this 70-year-old mistake. This is consistent with the fact [3] that some mathematical coordinate systems are incompatible with Einstein's equivalence principle[1)] and therefore not physically realizable (see also Appendix A). The belief that any coordinate system was valid as a space-time coordinate system in physics [4], has led Ohanian and Ruffini [5] to develop their gauge theory approach from which they criticize Einstein's theory [6,7] as premature. Here, it will be shown that their approach is not valid in physics.

The non-linear Einstein's field equation [6-8] reads

$$G_{ab} \equiv R_{ab} - \frac{1}{2} g_{ab} R = - K T_{ab} , \qquad (1a)$$

where G_{ab} is the Einstein tensor, R_{ab} is the Ricci curvature tensor, T_{ab} is the energy-stress tensors for matter, and K (= $8\pi\kappa c^{-2}$, and κ is the Newtonian coupling constant) is the coupling constant[2)].

However, the source term KT_{ab} generally depends on the space-time metric g_{ab} [8,9]. Thus, unlike the case of electromagnetism, this source term may not be considered as the cause of gravity represented by metric g_{ab} [10,11]. It follows that to consider gravity in a complete analogy manner with electromagnetism, as Ohanian and Ruffini [5] proposed, may not be valid (see also section 3). It should be noted in particular that the notion of gauge in gravity, though somewhat similar in mathematics, is physically different from the gauge notion of electromagnetism.

The harmonic coordinate condition [2,4-9] is

$$\frac{\partial}{\partial x^a} \left(|g|^{\frac{1}{2}} g^{ab} \right) = 0, \qquad (1b)$$

where g is the determinant of the metric. It was believed that a physical coordinate system with condition (1b) could be achieved by making the following gauge transformation, in terms of the covariant derivations,

$$\gamma_{ab} \rightarrow \gamma_{ab} + \nabla_a \zeta_b + \nabla_b \zeta_a \tag{1c}$$

where $\gamma_{ab} \equiv g_{ab} - \eta_{ab}$ and η_{ab} is the flat metric. Formally, the required gauge vector ζ_a would be obtained by solving the differential equation derived from Eq. (1b) and transformation (1c) (see section 2).

However, depending on the situations, a mathematical condition may be invalid in physics. (e.g., the spherical symmetry is invalid for a cubical massive source.) A counter example will be provided to illustrate that, for some problems, the "gauge condition" (1b) may not be applicable to obtain a physical coordinate system.

It should be pointed out that the question of applicability of (1b) is distinct from the Cauchy problem. In a Cauchy problem, the initial satisfaction of (1b) is assumed. In the validity problem of a gauge, a solution is given and then it is asked whether a gauge transformation exists such that the "gauge condition" (1b) becomes valid. Thus, although the existence of a mathematical solution for a Cauchy problem has been confirmed by Bruhat [12], the applicability problem of (1b) remains. On the other hand, since the harmonic coordinate condition (1b) is not applicable for some situations, a Cauchy solution may not be physically valid.

To establish a counter example, it is useful to consider the weak gravitational waves. In particular, an order counting in terms of the coupling constant is possible for weak gravity although, for a non-linear equation, a perturbative approach (e.g., linearization) may not necessarily be valid [13].

2. A Gauge Invariant Criterion.

For the purpose of examining weak gravity, it is convenient to use the notation in linearized gravity [4-9], which expresses relations in terms of deviations γ_{ab} ($\equiv g_{ab} - \eta_{ab}$) from the flat metric η_{ab}. Then, if only the first order terms are considered, eqs. (1a), (1b), and (1c) are respectively reduced to [4-6]

$$G^{(1)}{}_{ab} \approx -KT_{ab}, \qquad \text{where} \quad G^{(1)}{}_{ab} \equiv G_{ab} - G^{(2)}{}_{ab} \tag{2a}$$

and

$$G^{(1)}{}_{ab} = \frac{1}{2}\partial_c\partial^c \bar{\gamma}_{ab} + H^{(1)}{}_{ab}, \quad H^{(1)}{}_{ab} \equiv -\frac{1}{2}\partial^c[\partial_b \bar{\gamma}_{ac} + \partial_a \bar{\gamma}_{bc}] + \frac{1}{2}\eta_{ab}\partial^c\partial^d \bar{\gamma}_{cd};$$

and

$$\partial^{c}\bar{\gamma}_{cb} \approx 0 , \tag{2b}$$

where

$$\bar{\gamma}_{ab} \equiv \gamma_{ab} - \frac{1}{2}\gamma\,\eta_{ab} , \qquad \text{and} \qquad \gamma = \eta^{cd}\gamma_{cd} ;$$

and

$$\gamma_{ab} \rightarrow \gamma_{ab} + \partial_a\zeta_b + \partial_b\zeta_a , \tag{2c}$$

Then, using gauge (2b), the equation for ζ_a is reduced to

$$\partial^{c}\partial_{c}\zeta_b = -\,\partial^{c}\bar{\gamma}_{cb} , \tag{2d}$$

and

$$\frac{1}{2}\partial_c\partial^{c}\bar{\gamma}_{ab} = -KT_{ab} , \tag{2e}$$

is obtained from Eq. (2a). A crucial problem is whether (2d) has a solution, which is valid in physics. For the counter example in Section 4, Eq. (2d) has no physical solution and (2e) is not valid. One might argue, however, that gauge (1b) may still be valid since Eq. (2d) is only an approximation.

Thus, one has to develop a *non-perturbative* approach for weak gravity. If (2b) is valid, $G^{(1)}_{ab} \approx (1/2)\partial_c\partial^{c}\bar{\gamma}_{ab}$. This means that G_{ab} is of second order deviations if $\partial^{c}\partial_{c}\bar{\gamma}_{ab} \approx 0$ everywhere. It follows that if condition (1b) is valid, then $G^{(1)}_{ab}$ is of second order deviations since (2b) is the linear approximation of (1b). Thus, (1b) also implies that G_{ab} is of second order deviations. In conclusion, for weak waves ($\partial_c\partial^{c}\bar{\gamma}_{ab} \approx 0$ everywhere), "gauge condition" (1b) (or [2b]) is valid only if the related Einstein tensor is of second order deviations.

However, the above conclusion is a criterion only if the order of the Einstein tensor is invariant under a gauge transformation (1c). It is known that $G^{(1)}_{ab}$ is invariant under the linearized form of gauge transformation (2c) [2-6]. It fol-

lows that a transformation (1c) can change $G^{(1)}{}_{ab}$ only up to the second order deviations if the weak gravity condition is maintained. Thus, for weak gravity, the order of G_{ab} is invariant under a gauge transformation (exact and linearized), and G_{ab} is of either first or second order only. Indeed, for weak waves, the order of Gab is invariant.

3. Linear Field Equations and the Question of Gauge.

Due to inadequate understanding of Einstein's equivalence principle [7], many theorists, including the Wheeler-Hawking school [9,14], believed in unrestricted covariance. This misconception [3], in turn, is the theoretical basis of the current gauge notion. The so-called gauge freedom is due to the fact that there are only six independent equations from (1a) because of the Bianchi identities $\nabla^c G_{cb} = 0$. The additional four conditions to determine the metric g_{ab} is called the gauge condition. Since g_{ab} must satisfy Einstein's equivalence principle, these gauge conditions cannot be arbitrary [3]. Nevertheless, it was generally believed that the harmonic gauge can always be satisfied [4,5,8,9] although Einstein [7] has expressed some doubt on its general validity.

In case of weak gravity, a superficial support for the unrestricted gauge notion is that equation (2e) is similar to the Maxwell equation, and the harmonic gauge takes the form (2b), which is similar to the Lorentz gauge. These mathematical similarities are part of the motivation that leads Ohanian & Ruffini [5] to propose that the linear field equations can be based on their "gauge theory". Since they considered Einstein's theory as premature and not logical enough[3)] [5], they should have done better. The fact is, however, that their logic cannot be classified as valid.

They started with the notation h^{ab} for the gravitational field tensor, and they believed that the most general field equation that is linear in h^{ab}, is of second differential order, and contains T^{ab} as source must have the form,

$$\partial_c \partial^c h^{ab} + \alpha \partial^a \partial^b h^c_c - \alpha' (\partial_c \partial^b h^{ac} + \partial_c \partial^a h^{bc}) + \beta \eta^{ab} \partial_c \partial^c h^d_d + \beta' \eta^{ab} \partial_c \partial_d h^{cd} = -\kappa T^{ab}, \quad (3a)$$

where α, α', β, β', and κ are constants. It seems that, for the justification of their approach, there nothing more than personal preference and the speculation based on mathematical similarity. Also, based on the conservation law,

$$\nabla_a T^{ab} = T^{ab}{}_{;a} = \frac{\partial T^{ab}}{\partial x^a} + \Gamma^a{}_{ac} T^{cb} + \Gamma^b{}_{ac} T^{ac}, \quad (4)$$

$\kappa\partial_a T^{ab}$, is known to be of the second order. Then, Ohanian and Ruffini obtain the field equation

$$\partial_c\partial^c h^{ab} + \partial^a\partial^b h^c_c - (\partial_c\partial^b h^{ac} + \partial_c\partial^a h^{ac}) - \eta^{ab}\partial_c\partial^c h^d_d + \eta^{ab}\partial_c\partial_d h^{cd} = -\kappa\, T^{ab}\,, \qquad (3b)$$

and to their satisfactory, (3b) is invariant with respect to a gauge transformation

$$h^{ab} \to h^{ab} + \partial^a\Lambda^b + \partial^b\Lambda^a\,, \qquad (2c')$$

for any gauge vector Λ^a (even it is large). Moreover, by using $\partial_c\bar{h}^{cb} = 0$, one can reduce (3b) to

$$\partial_c\partial^c\bar{h}^{ab} = -\kappa\, T^{ab}\,, \qquad \text{where} \quad \bar{h}^{ab} \equiv h^{ab} - \frac{1}{2}\eta^{ab}h^c_c \qquad (3c)$$

Thus, their gauge theory approach could be valid although Newton's theory of gravity is not a gauge theory.

However, an implicit assumption in this approach is that a first order solution of gravity exists for a dynamic situation that produces gravitational radiation. Thus, it is necessary to check whether the resulting equation is compatible with this assumption. But, Ohanian and Ruffini [5] neglected this crucial step. Nevertheless, they later identified

$$\kappa h^{ab} = \eta^{ac}\gamma_{cd}\eta^{db}\,,$$

then

$$G^{(1)}{}_{ab} = -\frac{1}{2}\kappa^2 T_{ab}. \qquad (3d)$$

is equivalent to (3b). But, the implication of Eq. (3d) is distinct form that of Eq. (2a), which is an approximation. Whereas (2a) implies only that $K\partial_a T^{ab}$ is of second order; (3d) implies $\partial_a T^{ab} = 0$. Thus, when the energy tensor is $T^{\nu\mu}$ (m) of massive matter, as pointed out by Wald [4] and Yu [15], there is no gravitational radiation. Thus, (3b) is incor-

rect for massive matter in a dynamic situation. It thus follows that the derivation of linear gravity by Ohanian and Ruffini from their gauge theory is invalid. In the 80s, Logunov and Mestvirishvili [16] also criticized Einstein's theory from the viewpoint of a gauge approach, and it turns out that their theory has been found to be incorrect [17].

The harmonic gauge (2b) transforms (3d) to (2e) that allows radiation but is not gauge invariant. Moreover, as shown by Eddington [18], (2e) implies (2b) and is compatible with (4). Thus, Eq. (3) is just irrelevant [17]. This is consistent with our earlier analysis that a gauge condition cannot be arbitrary [3]. In a nonlinear equation, unlike the linear equation, the higher order terms can play a crucial part that Ohanian and Ruffini [5] over looked.

4. A Counter Example for the "Gauge Condition".

The above analysis has shown clearly that linear gravity is not a gauge theory. It is also known [13,17] that linearized gravity [2,4,5,9] is unreliable. Having established a simple criterion in section 2, it remains to identify a gravitational plane wave whose Einstein tensor is of first order deviations. The given counter example of plane waves would unambiguously show that some implications of linearized gravity are false. Let us consider a source tensor,

$$T_{ik} = P_i P_k\, g^{mn} A_m A_n - g^{mn}(\nabla_i A_m - \nabla_m A_i)(\nabla_k A_n - \nabla_n A_k), \tag{5a}$$

where

$$P_x = P_y = 0\,, \qquad P_t = -P_z = \omega\,, \tag{5b}$$

and ω is a constant [19]. Although what matter in here is whether the resulting metric gives a Lorentz manifold, it would be beneficial to clarify some theoretical considerations before hand. A_m represents an electromagnetic plane-wave propagating in the z-direction, and polarized diagonally in the x-y plane. Its non-zero components are:

$$A_x = A_y = \frac{A}{\sqrt{2}} \cos \omega(t - z), \tag{5c}$$

where A is a constant. Note that since P_m is a constant vector, $\omega(t - z) = P_m x^m = \int P_m dx^m$ represents a scalar. Then a solution[4)] of the Einstein field equation is

$$\gamma_{xx} = \gamma_{yy} = \gamma_{xy} = -\frac{KA^2}{4}[1 - \cos 2\omega(t - z)] , \tag{6a}$$

$$\gamma_{tt} = -\gamma_{zz} = [1 - 2\gamma_{xx}]^{-1/2} - 1 , \tag{6b}$$

and otherwise $\gamma_{ab} = 0$. Note that, from wave (6), both γ_{tt} and $(\gamma_{xx} + \gamma_{yy})$ are of first order deviations.

Thus, wave (6) has a polarization, which is linearly independent of the two polarizations allowed by linearized gravity [2,4,5,9]. Thus, wave (6) illustrates that implications of the linearized gauge on plane waves may not be valid.

This can also be shown explicitly. Solution (6) satisfies g = -1,

$$g^{tt} - 2g^{tz} + g^{zz} = g_{tt} + 2g_{tz} + g_{zz} = 0 , \tag{7a}$$

and

$$g^{xt} - g^{xz} = g^{yt} - g^{yz} = g_{xt} + g_{xz} = g_{yt} + g_{yz} = 0 . \tag{7b}$$

Relation (7) implies

$$\partial_a g^{ax} = \partial_a g^{ay} = 0 \quad \{\partial^a \bar{\gamma}_{ax} = \partial^a \bar{\gamma}_{ay} = 0\} , \tag{8a}$$

and

$$\partial_a(g^{at} - g^{az}) = 0 \qquad \{\partial^a(\bar{\gamma}_{at} + \bar{\gamma}_{az}) = 0\} , \tag{8b}$$

but

$$\partial_a(g^{at} + g^{az}) = 2\,\partial_t(g_{tt} + g_{tz})^{-1} \qquad \{\partial^a(\bar{\gamma}_{at} - \bar{\gamma}_{az}) = \partial^t(\gamma_{xx} + \gamma_{yy})\} . \tag{8c}$$

Due to Eq. (8c), Eq. (1b) (or (2b)) is valid only if the momentum $P^k = 0$. This suggests that (1b) cannot be satisfied physically. In fact, there is no physical gauge transformation, which can make (1b) satisfied.

One might argue that Hilbert [2] has shown that Eq. (2d) would have a physical solution for a source of massive matter. However, for such a case, an implicit assumption in Hilbert's "proof" is that the term $\partial^c \bar{\gamma}_{cb}$ (of first order)

does not satisfy $\partial^c\partial_c[\partial^a \bar{\gamma}_{ab}] \approx 0$ everywhere. This condition is not satisfied by metric (6). Also, Eq. (2e) does not provide an approximation for wave (6).

To confirm that the "gauge condition" is not applicable to the weak plane wave (6), one must prove additionally that its G_{ab} is of first order deviations. It follows Eq. (5) and Eq. (6) that

$$T_{ik} = \rho P_i P_k \,, \tag{9a}$$

where

$$\rho(t - z) = -\frac{A^2}{G}\cos[2\omega(t - z)] \,, \qquad \text{and} \qquad G = (1 - 2\,\gamma_{xx}). \tag{9b}$$

The source term KT_{ik} and therefore the Einstein tensor is of first order deviations. Thus, unless weak gravity depends on the chosen gauge, condition (1b) (or (2b)) is not a gauge that can be applied unconditionally.

It should be noted that not only, as Eq. (1a) implies, $\nabla_k T^{ki} = 0$ but also

$$\partial_k T^{ki} = 0. \tag{10}$$

Thus, the linearized conservation law is a necessary but not a sufficient condition for the "gauge" condition (2b). Eq. (10) implies that there is no gravitational acceleration for the photons [10,11,19].

One can see also that linearized gravity is not valid from the viewpoint of field equations. Although the source tensor (5) is non-zero, the metric elements (6) satisfy a homogeneous Maxwell equation

$$\partial_c\partial^c \bar{\gamma}_{ab} = 0 \,. \tag{11}$$

However, the linearized equation is an inhomogeneous Maxwell equation with a source of first order deviations. Thus, an approximation of the exact solution cannot be obtained through a linearized equation.

5. Physical Conditions on a Metric.

From the above analysis, although it may be possible to have a mathematical coordinate transformation such that the gauge condition is satisfied, the transformed metric would be neither weak nor a plane-wave. If the transformed metric were physically valid, there would be a problem of self-consistency because, from the viewpoint of physics, weak gravity should be independent of the valid gauge chosen because of the restriction due to the plane-wave source.

It should be noted that not every solution of Einstein equation is valid in physics (Appendix A). A solution of Einstein's equation may not be compatible with Einstein's equivalence principle, which requires a time-like geodesic represents a physical free falling [7]. This means that all physical requirements must be satisfied. The mathematical existence of a free falling local Minkowski space is only a necessary condition. Thus, coordinate relativistic causality[5)] must be satisfied if the principle of equivalence is to be satisfied [20].

The metric of solution (6) has the following form:

$$ds^2 = (1 + 2\beta)^{-1/2}(dt^2 - dz^2) - \beta(dx + dy)^2 - dx^2 - dy^2 ,$$

where

$$\beta = \beta(u) = \frac{KA^2}{4}[1 - \cos 2\omega(t-z)] \geq 0. \qquad (12)$$

From $ds^2 = 0$, according to Einstein [7], one obtains the velocity of light in the x-direction is determined by

$$(dx/dt)^2 = (1 + 2\beta)^{-1/2}(1 + \beta)^{-1}. \qquad (13)$$

Since $\beta \geq 0$, Eq. (13) implies that $(dx/dt)^2 \leq 1$. In fact, metric (12) satisfies coordinate relativistic causality.

Satisfying coordinate relativistic causality would reject non-physical metric solutions [11] (such as those given by Peres [21] and Bonner [22]). Thus, the existence of a Cauchy solution does not imply that a physical gauge transformation exists for solution (6). From linearized gravity, one may obtain another type of solutions [23],

$$\bar{\gamma}_{tt} = \bar{\gamma}_{zz} = -\bar{\gamma}_{tz} = -\bar{\gamma}_{zt} = \frac{t+z}{4}\omega^2 \int_0^u f(t)dt ,$$

and

$$\bar{\gamma}_{xt} = -\bar{\gamma}_{xz} = \bar{\gamma}_{tx} = -\bar{\gamma}_{zx} = \frac{x}{2}\omega^2 \int_0^u f(t)dt \tag{14}$$

where f(u) = $2KA^2\cos 2\omega u$. Otherwise $\bar{\gamma}_{ab}$ is zero. However, these metrics are not bounded. They are not physically realizable because they do not satisfy relativistic causality. Solution (14) is unrelated to (6) by a physical gauge transformation (see also Appendix B).

Physically, solution (6) is the accompanying gravitational wave of electromagnetic wave (5c) [11,19]. This solution is consistent with special relativity since the flat metric is an accurate approximation if the electromagnetic wave is weak. Then, solution (6) has shown that the gauge is not always applicable. (Note that, in order to have a physical solution, it is necessary to include an energy-stress tensor for photons in source tensor (5) [11,19]). In current theory, the "proof" for the applicability of the "gauge" condition deals directly with the metric, but now we have shown that the nature of source tensor is also important.

6. Conclusion and Discussion.

In conclusion, the new approach of Ohanian and Ruffini, based on their notion of gauge invariance, has been shown to be invalid. The root of their problem is due to inadequate understanding of Einstein's equivalence principle. This is perhaps also the reason that Einstein's equivalence principle is ignored in their book. Understandably, due to their misconception on general relativity, Einstein's theory would appear to them to be insufficiently logical.

In general relativity, the gauge notion is inadequate because the restriction by Einstein's equivalence principle has not been included. The "gauge condition" (1b) (or [2b]) is not unconditionally applicable to obtain a physical coordinate system. This conclusion is independent of whether a perturbative approach is valid. The given example shows also that solutions of a non-linear equation may not be approachable by the usual linearized perturbation.

Moreover, Eq. (2b) also implies the linearized energy-momentum conservation law, which is the root for Einstein's radiation formula to be criticized as not self-consistent [4,16]. Since Einstein's radiation formula is supported by observations [24], the gauge condition (2b) is not generally applicable. Note that, in electrodynamics, the classical gauge invariance is also proven to be inconsistent with experiment [25-27].

In this paper, it has been shown also that, in theoretical physics, it is important to go through concrete details (see also Appendices A & B). Also, it would be helpful to remember that, depending on the physical situation, a mathe-

matical field equation does not necessarily imply the existence of a physical solution. A solution in physics must additionally satisfy related physical requirements such as coordinate relativistic causality and the principle of causality[6].

Physically, the existence of gravitational waves is due to the fact that gravitational influence must propagate with a finite speed [28,29]. Thus, logically the existence of gravitational waves is independent of the "gauge condition". Exact solution (6) is an example that the existence of gravitational waves is independent of linearized gravity. For plane waves, equation (11) implies that a plane wave travels at the speed of light; whereas condition (1b) would exclude some possible polarizations.

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Appendix A: "Gauge" and the Equivalence Principle.

Solution (6) shows that the harmonic "gauge" has not been really established. Thus, the current notion of gauge is a result of "formal" mathematics, which is defined as doing "mathematical" manipulations without verifying the related mathematical and physical validity. Here, the problems in formal mathematics shall be further illustrated. In particular, the current notion of gauge can be incompatible with Einstein's equivalence principle.

Currently, the notion of gauge is based on the mathematical concept, diffeomorphism, which is a one-one, onto, C^∞ (infinitely differentiable) map between two manifolds and its inverse map is C^∞ [4]. This notion is based on the fact that a diffeomorphism preserves the tensor calculation operations. However, this notion of gauge is inadequate since the choice of physical coordinates is also restricted by Einstein's equivalence principle. Some mathematical coordinate transformations are invalid in physics because they may not be realizable.

The valid choice of a coordinate is in a confusing state among some current theorists. For instance, Hawking [30] made clear in 'The Arrow of Time' of his book that a time coordinate is, in several aspects, distinct from a space coor-

dinate. Nevertheless, he also wrote in the same book that "there is no real distinction between the space and time coordinate, just as there is no real difference between any two space coordinates"!

As Einstein [31] pointed out, "The proposition of mathematics referred to objects of our imagination, and not to objects of reality." A general problem among some relativists is that they are unable to distinguish mathematics from physics. In particular, when a transformation mixes both the time coordinate and a space coordinate, examining the physical validity of such a transformation is advised.

It is known that Galilean transformations are incompatible with special relativity. It will be shown that the resulting coordinate system is incompatible with the equivalence principle. Let us consider a Galilean transformation,

$$x' = x, \quad y' = y, \quad t' = t, \quad \text{and} \quad z' = z + vt. \tag{A1}$$

In the new coordinate system, the z'-direction forward light speed is

$$dz'/dt' = dz/dt + v = c + v, \quad \text{and} \quad dz'/dt' = -c + v \tag{A2}$$

for the backward z'-direction. Nevertheless, Galilean transformation (A1) is a diffeomorphism and thus a valid gauge in current notion. Moreover, (A1) transforms the Minkowski metric, $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ to

$$ds^2 = c^2dt'^2 - (dz' - vdt')^2 - dx'^2 - dy'^2 \tag{A3}$$

for a system K'. The above metric illustrates that $ds^2 = 0$ does not provide a criterion for the validity of a coordinate transformation; whereas a crucial physical requirement is that the light speed c in vacuum is the maximum speed.

Metric (A3) illustrates that Einstein's equivalence principle is applicable only to physically realizable metrics. Metric (A3) is a constant, and there is no gravitational acceleration. Thus, a free falling non-rotating frame can have only a constant speed. Hence, a local coordinate system is obtained only through a Lorentz coordinate transformation. But, a Lorentz transformation cannot transform metric (A3) to a local Minkowski space. Thus, the current notion of gauge can be incompatible with the equivalence principle.

The equivalence principle is based on the local equivalence between acceleration and gravity. However, many as Yu [32, p. 42] incorrectly believed that Einstein's principle is satisfied if at any space-time point, it is always possible to establish a local Minkowski space., which is related to a "free falling". But this is only necessary but insufficient.

Nevertheless, mathematics ensures the existence of a local Minkowski space for a free falling. Now, consider an observer P' resting at (x', y', z', t'). The geodesic is

$$dx'/ds = dy'/ds = dz'/ds = 0, \qquad \text{and} \qquad dt'/ds = (c^2 - v^2)^{-1/2} \tag{A4}$$

Accordingly, (0,0,0,dt') is the local time coordinate of P'. Then, the orthonormal vectors of the tetrad are

$$a_1 = (1,0,0,0), \quad a_2 = (0,1,0,0), \quad a_3 = (0,0,\alpha,\beta), \quad \text{and} \quad b_{p'} = (0,0,0,\gamma) \tag{A5}$$

where

$$\alpha = \gamma^{-1}, \ \beta = -\gamma v c^{-2}, \qquad \text{and} \qquad \gamma = (1 - v^2c^{-2})^{-1/2}.$$

The corresponding transformations is as follows:

$$dt' = \gamma\,(dT - vc^{-2}dZ), \qquad dz' = \gamma^{-1}dZ, \tag{A6}$$

and

$$dx' = dX, \qquad \text{and} \qquad dy' = dY.$$

Thus, (dx', dy', dz') and (dX, dY, dZ) share the same frame of reference because there is no acceleration nor relative motion. But, there is spatial measurement changes in the z-direction. Metric (A3) does not satisfy the equivalence principle since there is no physical cause for transformation (A6). In relativity, such a physical transformation happens only when there is motion and/or acceleration. Thus, geodesic (A4) does not represent a physical free fall.

The Galilean transformation is an unphysical transformation, and it simply takes another unphysical transformation to cancel out the unphysical properties so introduced. In fact, (A1) and (A6) imply

$$dt = \gamma\,(dT - vc^{-2}dZ), \qquad \text{and} \qquad dZ = \gamma\, dz' = \gamma(dz + v\, dt). \tag{A7}$$

Transformation (A7) is just a Lorentz-Poincaré, transformation.

To circumvent whether a coordinate system is physically valid, some relativists "define" the light speed at any point of a manifold in terms of a local Minkowski space. (Note that although a local Minkowski space may exist, as mentioned above, the transformation may not be due to a free falling.) However, such a "definition" is not well defined since there are manifolds, which cannot be diffeomorphic to a physical space[7]. Thus, this "definition" is also misleading in physics since satisfying the equivalence principle must be implicitly assumed.

To illustrate the existence of an intrinsic non-physical space, let us consider the following metric [33],

$$ds^2 = du\, dv + h_{ij}(u)x^i x^j du^2 - dx^i dx^i, \qquad \text{(A8)}$$

where $u = ct - z$, $v = ct + z$, $h_{ii}(u) \geq 0$, and $h_{ij} = h_{ji}$. This metric satisfies the harmonic gauge.

It will be shown that metric (A8) is not compatible with coordinate relativistic causality. Metric (A8) has a form,

$$ds^2 = du\, dv + H du^2 - dx^i dx^i\ . \qquad \text{(A9)}$$

A light trajectory satisfies $ds^2 = 0$. For a light in the z-direction (i.e. $dx = dy = 0$), one obtains

$$ds^2 = du dv + H du^2 = 0, \quad \text{and} \quad \frac{dz}{dt} = -c\frac{1+H}{1-H} \qquad \text{(A10)}$$

is the backward light speed while the forward light speed is c. Then, coordinate relativistic causality requires $H \leq 0$. Obviously, this condition is not satisfied by metric (A8).

Metric (A8) depends on x (= x^1) and y (= x^2), and is not weak nor bounded although the cause of metric (A8) can be an electromagnetic plane wave. Metric (A8) satisfies

$$\eta^{ab}\partial_b\partial_a\gamma_{tt} = -2[h_{xx}(u) + h_{yy}(u)\ , \quad \text{and} \quad \gamma = \eta^{cd}\gamma_{cd} = 0, \qquad \text{(A12)}$$

where γ_{ab} is a deviation from η_{ab}. Thus, metric (A8) is also arbitrary as those derived by Peres [21] and Bonnor [22].

The gravitational force is related to $\Gamma^z_{tt} = (1/2)\partial H/\partial t$. There are arbitrary non-physical parameters (the choice of origin), which are not related to the cause (an electromagnetic plane wave). In other words, the principle of causality is also violated. Such a violation is intrinsic, since a diffeomorphism cannot transform away an unphysical parameter. Thus, it is impossible to transform this metric to a physical space, which satisfies the equivalence principle.

Appendix B: An Example of Unphysical "Gauge" Transformation

As pointed out by Einstein [18], the notion of weak gravity may not be maintained. To illustrate the non-physical connection between metrics (12) and (14), let us consider the following change in "coordinates" [23]:

$$X = \frac{1}{\sqrt{2}}(x + y)(1 + \beta), \quad \text{where} \quad \beta = \frac{KA^2}{4}[1 - \cos 2\omega(t-z)] \geq 0$$

$$Y = \frac{1}{\sqrt{2}}(x - y),$$

$$Z = \frac{1}{2}(t + z)(1 + 2\beta)^{-1/2} - \frac{1}{2}(t - z),$$

$$T = \frac{1}{2}(t + z)(1 + 2\beta)^{-1/2} + \frac{1}{2}(t - z). \qquad \text{(B1)}$$

Since u = t - z = T - Z, there is no change for the electromagnetic plane wave (5). Then, metric (12) is transformed to

$$ds^2 = dT^2 - dZ^2 - dY^2 + (1 + 2\beta)^{-1}(Z + T)d\beta(dT - dZ) - (1 + \beta)^{-2}(1 + 2\beta)\,[dX - X(1 + \beta)^{-1}d\beta]^2 \qquad \text{(B2)}$$

For $A = 0$, (B1) reduce to a rotation. Now approximate, using $\beta << 1$, and so obtain

$$ds^2 \cong dT^2 - dZ^2 - dY^2 - dX^2 + \frac{d\beta}{du}[(Z+T)(dT - dZ)^2 + 2XdX(dT - dZ)] \qquad \text{(B3)}$$

where

$$\frac{d\beta}{du} = \frac{KA^2}{2}\sin(2\omega u).$$

Note that (B3) is not yet (14). Now consider the gauge vector ζ_i.

$$\zeta_X = \zeta_Y = 0;$$

$$\zeta_Z = \lambda[\sin 2\omega(T - Z) + 2\omega\,(T + Z)\cos 2\omega(T - Z)]\,;$$

$$\zeta_T = \lambda[\sin 2\omega(T - Z) - 2\omega(T + Z)\cos 2\omega(T - Z)]\,. \qquad \text{(B4)}$$

For weak gravity, if $\lambda = -KA^2/32\omega$, then (B4) transforms (B3) to metric (14). The fact that the term of the form $f(T - Z)(T + Z)[dT - dZ]^2$ can be "gauged" away, manifests the arbitrary nature of this calculation. Now it seems that metric (12) could be transformed to (14).

However, one should examine whether transformations (B1) and (B4) are physically valid. First, because of the factor (T + Z), the gauge vector ζ_i may not be small and therefore the linearized gauge transformation may not be adequate. Moreover, (B4) transforms a bounded metric (12) to an unbounded metric (B2). Since weak gravity, a physical condition, should be independent of the coordinate chosen, (B1) may not be a physically valid coordinate transformation. In other words, X, Y, Z, and T may not be a valid coordinate system.

It will be shown that although the electromagnetic plane wave (5) has the same form after transformation (B1), the physics of the space has profoundly changed even though β is very small. From (B1), one obtains

$$dX = \frac{1}{\sqrt{2}}(x + y)\beta' du + \frac{1}{\sqrt{2}}(dx + dy)(1 + \beta),$$

$$dY = \frac{1}{\sqrt{2}}(dx - dy),$$

$$dZ = \frac{1}{2} dv(1 + 2\beta)^{-1/2} - \frac{1}{2}(t + z)(1 + 2\beta)^{-3/2}\beta' du - \frac{1}{2} du,$$

$$dT = \frac{1}{2} dv(1 + 2\beta)^{-1/2} - \frac{1}{2}(t + z)(1 + 2\beta)^{-3/2}\beta' du + \frac{1}{2} du, \qquad \text{(B5)}$$

For a forward light located at the origin of an x-y plane, du = 0. It follows from (B5) that dZ/dT = 1 as expected. But, for a backward light (dx = dy = 0, dz/dt = -1) located at the origin of an x-y plane, dv = 0. It follows from (B5)

$$dX = dY = 0,$$

and

$$dZ/dT = \{\beta'(t + z)(1 + 2\beta)^{-3/2} + 1\}/\{\beta'(t + z)(1 + 2\beta)^{-3/2} - 1\} \neq -1 \qquad \text{(B6)}$$

Thus, in the forward direction, the speed of light is the usual constant. However, in the backward direction (unless β' = 0), the light velocity is not a constant, but depends on (t + z). (Thus, the light speed can be zero or reach infinite!) From (B6) the deviation of dZ/dT from -1 can be very large. This is not consistent with the assumption that the light speed in vacuum is the maximum speed. This means that Einstein's equivalence principle is not compatible with this metric. Thus, (B1) is not valid in physics, and the coordinates, X, Y, Z, T, do not form a physical coordinate system.

One might argue that, if the speed of light is "defined" in a local coordinate system, light speed is always c. However, as pointed out earlier, such a definition of light speed in a manifold is not well defined. Moreover, a speed in a local coordinate system is not a speed in the manifold. Einstein [34] pointed out that "The principle of inertia and the principle of the constancy of the velocity of light are valid only with respect to an inertial system."

For a physical coordinate system, it should be physically realizable. As pointed out by Einstein [35], "In physics, the body to which events are spatially referred is called the coordinate system." However, if a coordinate transformation is considered only as a mathematical tool, then the only requirement is that the transformation is one-one and sufficiently smooth. While such a mathematical tool may have some advantages in simplifying the mathematics, such a mathematical coordinate system may not have a valid physical meaning. Currently, however, the physical meaning of such a coordinate system is invalidly assumed to be unconditionally valid.

ENDNOTES

1) Einstein's equivalence principle requires that a free fall must result in a co-moving local Minkowski space. Einstein regards that the resulting local space being Minkowski is a physical must, whereas Pauli's version regards it as only a mathematical possibility. Thus, according to Einstein, the causes and consequences of such local coordinate transformation must be investigated in terms of physics, whereas they are ignored in Pauli's version [1,2].

2) General relativity, based on Einstein's equivalence principle and the principle of general relativity, was proposed by Einstein [6] in 1916 and subsequently modified [7] in 1921 because of a now obvious inconsistency between physics and unrestricted covariance, which is an extension of the principle of general relativity as an interim assumption [6]. Thus, covariance must be compatible with his equivalence principle, which the Wheeler-Hawking school implicitly replaced [36] with the existence of local Minkowski space. The 1915 Einstein equation was inconsistent with binary pulsar experiments and the 1995 update of Einstein equation was proposed [17] that has been proven to be compatible with general relativity [37]. Also, some authors [5] prefer, different from Einstein [7], to define $K = 8\pi \kappa c^{-4}$. Then, the four velocity u^{μ} would be defined as $c dx^{\mu}/ds$, where $ds^2 = g_{\alpha\beta}\, dx^{\alpha} dx^{\beta}$. However, Eq. (1a) remains the same.

3) Ohanian [38] would like to clarify himself as an admirer of Einstein, and kindly explained, "When we characterize Einstein's insight into the character of gravitation as premature, we don't mean this as a criticism, but as a compliment. Einstein was far ahead of the physicists of his time, and he managed to discover the theory of General Relativity before relativitistic field equations (and especially the equations for spin 2) and gauge theories were fully understood. For this he deserves our profound respect and admiration. The approach taken in the Ohanian-Ruffini book is an attempt to see if there is an alternative way to arrive at General Relativity, a way that would perhaps have been followed if Einstein had not taken his amazing shortcut to the final result." However, he *did not retract* their criticism toward Einstein's equivalence principle or the claim, "there is no such thing as a relativity more general than special relativity"

4) Because an electromagnetic plane wave is a valid idealization in physics, as pointed out by Misner et al [9], such a plane wave can be used as an idealized source in gravity. Due to inadequate understanding of special relativity, some theorists [39,40] would have objected this because the total energy of a plane wave is infinite. Different from

the energy of massive matter, the influence of the energy of an electromagnetic wave, which travels at the speed of light, is essentially local [11,19]. This is a physical reason that a bounded solution can be obtained.

5) Coordinate Relativistic Causality is a physical requirement that the possible maximum velocity of propagation is the speed of light in special relativity. Due to behaviour of rods and clocks under gravity, coordinate relativistic causality is strengthened as shown by Einstein's calculation of the bending of light-rays. Thus, coordinate relativistic causality is a useful tool to reject unphysical coordinate system, which could be mistaken as a valid space-time coordinate system. However, some theorists rejected this because they believed that space-time coordinates should have no meaning in physics although this is in conflict with the fact that there are vectors in physics.

6) The time-tested assumption that phenomena can be explained in terms of identifiable causes is called the principle of causality. This is the basis of relevance for all scientific investigations. The principle of causality implies that any parameter in a physical solution must be related to some physical causes. In general relativity, Einstein and subsequent theorists have used this principle implicitly on symmetry considerations [2-11].

7) A physical space, which models reality such that all physical requirements are sufficiently satisfied, has a frame of reference that has the Euclidean structure and a time coordinate that is related to local clocks. Thus, the theoretical framework of general relativity defines the space-time coordinates in terms of physics.

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Résumé

Il est démontré que l'applicabilité de la jauge harmonique n'a jamais été généralement établie si l'on s'attend à ce que le système coordonné résultant soit physiquement réalisable. On donne une onde plane gravitationnelle pour montrer que la "jauge" harmonique peut ne pas être applicable et que les implications de la jauge linéarisée sur les ondes planes

peut être invalide. Concurremment, il est démontré que pour des ondes gravitationnelles faibles, la jauge harmonique est valide seulement si le tenseur d'Einstein est en déviation de deuxième ordre de la métrique plane. Puisque tel ordre est invariant de jauge, une onde gravitationnelle faible avec un tenseur d'Einstein de premier ordre implique que cette jauge harmonique peut être mal appropriée. De plus, il est démontré que la "condition de jauge" peut ne pas être compatible avec la causalité relativiste coordonnée et le principe d'équivalence. Concurremment, il est aussi démontré que l'approche de Ohanian et de Ruffini, dans laquelle, à la différence de celle d'Einstein, la gravité linéaire est considérée comme étant une dérivée de leur théorie de jauge, n'est pas valable en physique.